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Dynamic Behavior Of Torsionally Coupled Buildings For Foundations Embedded In An Elastic Stratum

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ABSTRACT: The purpose of this work is to investigate the results of deformability of the soil medium which radiates energy, on the coupled lateral-torsional dynamic response of single story buildings under seismic loading. An efficient method idealized the whole structure-foundation-soil system with a three dimensional lumped parameter model with frequency-dependent foundation impedance functions for the two cases of a site, surface and embedded footing-layer on rigid substratum is presented.

Keywords : Soil-Structure Interaction, Impedance Functions, Fast Fourier Transform, Embedded Footing, Layered Medium.

ÖZET: Bu çalışmada, planda simetriklik özelliği göstermeyen burulmalı bağlaşık tek katlı yapı ile zeminden oluşan ortak sistemin yer hareketi etkisi altında zorlanmış titreşimi için geliştirilen matematik model üzerinde üstyapı ile zemin arasındaki karşılıklı etki olayı incelenmiştir. Bu yaklaşımda, ana kaya üzerindeki tabakalı zemine oturan yüzeysel ve zemine gömülü rijit temeller için elde edilmiş frekansa bağlı empedans fonksiyonlarına kullanılmıştır.

Introduction

Most of the analysis of torsional coupling in both the elastic and inelastic range of earthquake response are limited to the case in which the structure is founded on an infinitely rigid foundation medium (Kan 1981 & Chandler 1986). However, the intensive research works and investigations of the effects of past earthquakes have found out that the elastic response of structures to earthquake ground excitations is influenced by deformability of the soil. Therefore, the rigid foundation assumption represents an approximation to the real conditions for the seismic design of buildings according to current building codes. The effects of soil-structure interaction (SSI) on the dynamic response of building systems have been subject of numerous investigations in recent years. Various analytical and numerical techniques were also proposed and developed to efficiently simplify the SSI analysis, such as transmitting boundries of different kinds, boundary elements, and infinite elements and their coupling procedures for modelling of unbounded media (Sivakumaran 1992, Meek 1992, Aydınoğlu 1993, Wolf 1996). However, the complicated formulation and intensive computation to obtain the exact solution for this problem restrict its common application to traditional engineering practice up to now.

The goal of this study is to evaluate the effect of three dimensional soil-structure interaction (SSI) on the elastic response of building-foundation systems under earthquake excitation for the two cases of a site, surface and embedded footing-layer on rigid substratum. In the present paper, therefore, an efficient methodology using lumped parameter model and accomplished in the frequency domain by using the fast Fourier transform algorithm to obtain the dynamic response of torsionally asymmetric buildings including soil-structure interaction effect is presented. The effects of the controlling parameters such a building eccentricity, depth of embedment, thickness of layer, length of lateral welded contact between soil and foundation, and excitation characteristics on the foundation impedance are also investigated to assess the dynamic behavior of the soil-structure system. The numerical results presented in this paper show that the dynamic response of a building-foundation system including soil-structure interaction can be significiantly different from that calculated with a fixed-base model.

System Model and Equations of Motion

Idealization of the Structure-Soil System

The building is modelled as an elastic single-story three dimensional structure resting on surface and embedded foundation, respectively as shown in Figure 1, consisting of shear frames, resting on a rigid square foundation of mass m_o with of negligible thickness. The mass of this building is considered to be concentrated at floor level, the floor system is assumed to be rigid rectangular floor deck supported by relatively massless, axially inextensible columns. The lateral load resisting element are assumed to be arranged so that the system has none axis of symmetry. Thus the structure is two-way torsionally coupled system with three degree of system, namely horizontal displacements u_x, u_y and rotation about vertical axis u_q. In addition, due to the deformability of foundation, the system has five more displacement degrees of freedom, namely horizontal translations of the foundation u_{ox}, u_{oy}, rocking rotations of the foundation g_{ox}, g_{oy} and the twist of the foundation q_o (Figure 1). The earthquake excitation is defined by $\ddot{u}_{gx}(t)$ and $\ddot{u}_{gy}(t)$, the x and y components of the ground acceleration, and $\ddot{u}_{gq}(t), \ddot{g}_{gx}(t), \ddot{g}_{gy}(t)$ the rotational acceleration of the base of the building about the vertical and horizontal axis respectively.

In this study, the soil comprises a layer of thickness H which is homogeneous and isotropic and rest on a rigid substratum and the foundation medium is modelled by equivalent springs and dashpots such that the resulting equations of motions are similar

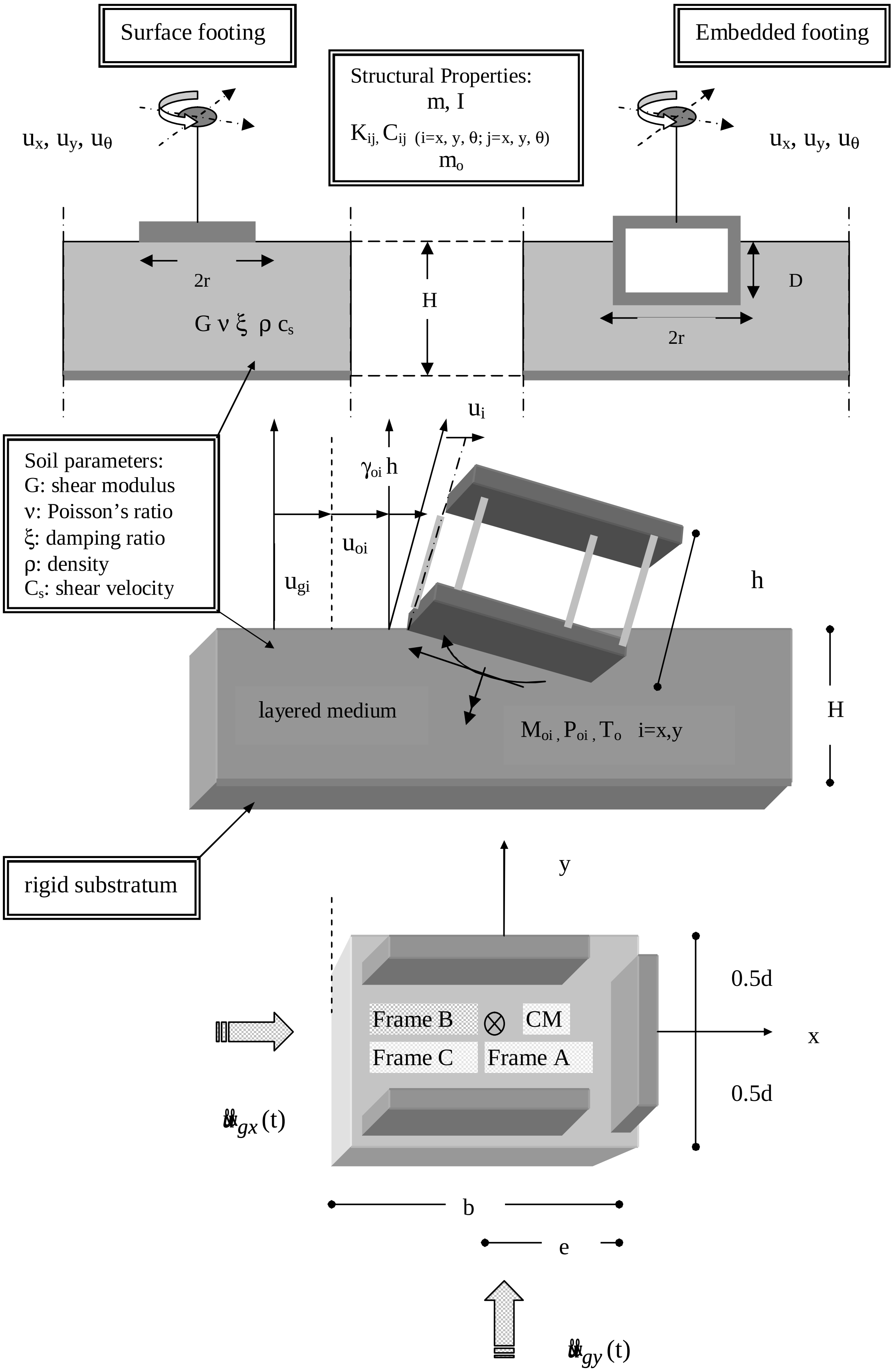

Figure 1. Idealization of Structure with Rigid Footing-Layer on a Rock substratum

those of a fixed-base structures. However, for accurate representation the elastic soil medium, the properties of springs and dashpots are requiered to be depended on the frequency of excitation. Thus, the governing equations for the structure-foundation system are expressed and solved in the Fourier transformed frequency domain. In this paper, the Fourier transform approach, in which the response is first evaluated in the frequency domain by using fast Fourier transform and then transformed into the time domain is used to obtain the structural response of torsionally coupled buildings including soil-structure interaction effects.

Analytical Procedures

The governing equations are developed considering the motion of the superstructure and the whole system. The procedure of this method, first of all the structural deformations are obtained in terms of foundation displacements which in combination with the dynamic soil-structure interaction force-displacement relationships depending on impedance functions proposed by Kausel (1974), Tassoulas (1981) and Apsel-Luco (1987). The equations of motion for a soil-structure interaction system with eight degrees of freedom can be written in the usual matrix form:

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I \end{bmatrix} \begin{Bmatrix} \ddot{u}_x(t) \\ \ddot{u}_y(t) \\ \ddot{u}_\theta(t) \end{Bmatrix} + \begin{bmatrix} C_{xB}+C_{xC} & 0 & (d/2)(C_{xB}-C_{xC}) \\ 0 & C_{yA} & eC_{yA} \\ (d/2)(C_{xB}-C_{xC}) & eC_{yA} & e^2C_{yA}+(d^2/4)(C_{xB}+C_{xC}) \end{bmatrix} \begin{Bmatrix} \dot{u}_x(t) \\ \dot{u}_y(t) \\ \dot{u}_\theta(t) \end{Bmatrix}$$

$$+ \begin{bmatrix} K_{xB}+K_{xC} & 0 & (d/2)(K_{xB}-K_{xC}) \\ 0 & K_{yA} & eK_{yA} \\ (d/2)(K_{xB}-K_{xC}) & eK_{yA} & e^2K_{yA}+(d^2/4)(K_{xB}+K_{xC}) \end{bmatrix} \begin{Bmatrix} u_x(t) \\ u_y(t) \\ u_\theta(t) \end{Bmatrix} \tag{1}$$

$$= -\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I \end{bmatrix} \begin{Bmatrix} \ddot{u}_{gx}(t)+\ddot{u}_{ox}(t)+h\ddot{\gamma}_{ox}(t) \\ \ddot{u}_{gy}(t)+\ddot{u}_{oy}(t)+h\ddot{\gamma}_{oy}(t) \\ \ddot{\theta}(t) \end{Bmatrix}$$

In addition five more equations are needed in order to completely solve the problem. These equations are developed by considering the equilibrum of the whole system. They can be further written in the open form as:

$$m_o(\ddot{u}_{oi}(t)+\ddot{u}_{gi}(t)) + m(\ddot{u}_{gi}(t)+\ddot{u}_{oi}(t)+h\ddot{\gamma}_{oi}(t)+\ddot{u}_i(t)) + P_{oi}(t)=0 \tag{2a}$$

$$I_{tx}\ddot{\gamma}_{oi}(t) + m\,h\,(\ddot{u}_{oi}(t)+h\ddot{\gamma}_{oi}(t)+\ddot{u}_i(t)) + M_{oi}(t)=0 \tag{2b}$$

$$m_o r_o^2\ddot{\theta}_o(t) + m r^2(\ddot{\theta}_o(t)+\ddot{u}_\theta(t)) + T_o(t)=0 \qquad (\text{for } i=x,y) \tag{2c}$$

If $P_{oi}(t)=\tilde{P}_{oi}\,exp(i\omega t)$, $M_{oi}(t)=\tilde{M}_{oi}\,exp(i\omega t)$ and $T_o(t)=\tilde{T}_o\,exp(i\omega t)$ are respectively the harmonic horizontal force, rocking moment and torque acting on a massless rigid circular disc supported on an elastic soil surrounded a layer of thickness H which rests on a rigid substratum, the steady-state response of the disc at frequency ω will be given by the harmonic displacements $u_{oi}(t)=\tilde{u}_{oi}\,exp(i\omega t)$, $\gamma_{oi}(t)=\tilde{\gamma}_{oi}\,exp(i\omega t)$ and $\theta_o(t)=\tilde{\theta}_o\,exp(i\omega t)$ for i=x,y respectively. The force-displacement relationship may be expressed as

$$\begin{Bmatrix} \tilde{P}_{ox} \\ \tilde{P}_{oy} \\ \tilde{M}_{ox} \\ \tilde{M}_{oy} \\ \tilde{T}_{o} \end{Bmatrix} = \begin{bmatrix} K_{hx}(w) & 0 & K_{hxrx}(w) & 0 & 0 \\ 0 & K_{hy}(w) & 0 & K_{hyry}(w) & 0 \\ K_{hxrx}(w) & 0 & K_{rx}(w) & 0 & 0 \\ 0 & K_{hyry}(w) & 0 & K_{ry}(w) & 0 \\ 0) & 0 & 0 & 0 & K_{t}(w) \end{bmatrix} \begin{Bmatrix} \tilde{u}_{ox} \\ \tilde{u}_{oy} \\ \tilde{g}_{ox} \\ \tilde{g}_{oy} \\ \tilde{q}_{o} \end{Bmatrix} \tag{3}$$

The numerical approximations of the impedance functions used in the following study are taken from (Sieffert 1992) in the case of surface and embedded footings. Assuming exterior terms of diagonal terms of dynamic stiffness matrix which introduces neglible errors for most pratical purposes, the impedance functions are described for the terms of the main diagonal of the matrix as:

$$K_j = K_{s\infty j}(k_j(a_o) + i a_o c_j(a_o))(1 + 2ib) \tag{4}$$

in which r is the radius of the foundation disc. The internal damping of soil is also taken into consideration and is characterized by the damping ratio β. In this complex-variable notation k_j represent the dimensionless spring coefficient and c_j the corresponding damping coefficient depending on a_o and the Poisson's ratio ν. Static stiffness coefficients may be defined (Gazetas 1983) as $K_{S\infty j}$ for each degree of freedom - or mode - (for index hx and hy; horizontal displacemets along the x and y axes, and for index rx, ry and t; rotations along x, y and z axes respectively). The soil-structure interaction relationships given by equation (3) were developed for a massless circular rigid disc. In this work, whereas, they have been carried out to a rectangular foundation in an approximate way by use of equivalent values for the radius of the rigid disc r_o such that the resulting static stiffness coefficients are the same as those corresponding to rectangular foundation (Thomson 1963). Due to these frequency based expressions for interaction shear force, overturning moments and torque, the interaction problem lends itself readily for formulation in the frequency domain. To do so, the Fourier transform is applied to the equations of motion (1) and (2a-c), Then equation (3) are substituted into these transformed equations(2a-c). The structural displacements can be eliminated from the resulting equations. It can be written only in terms of foundation displacements (Çelebi 2001).

$$\begin{bmatrix} a_{11}(\omega) & a_{12}(\omega) & a_{13}(\omega) & a_{14}(\omega) & a_{15}(\omega) \\ a_{21}(\omega) & a_{22}(\omega) & a_{23}(\omega) & a_{24}(\omega) & a_{25}(\omega) \\ a_{31}(\omega) & a_{32}(\omega) & a_{33}(\omega) & a_{34}(\omega) & a_{35}(\omega) \\ a_{41}(\omega) & a_{42}(\omega) & a_{43}(\omega) & a_{44}(\omega) & a_{45}(\omega) \\ a_{51}(\omega) & a_{52}(\omega) & a_{53}(\omega) & a_{54}(\omega) & a_{55}(\omega) \end{bmatrix} \begin{Bmatrix} \tilde{\theta}_{o}(\omega) \\ \tilde{u}_{ox}(\omega) \\ \tilde{u}_{oy}(\omega) \\ \tilde{\gamma}_{ox}(\omega) \\ \tilde{\gamma}_{oy}(\omega) \end{Bmatrix} = \begin{Bmatrix} b_1(\omega) \\ b_2(\omega) \\ b_3(\omega) \\ b_4(\omega) \\ b_5(\omega) \end{Bmatrix} \tag{5}$$

When the displacements of foundation are determined in frequency domain, the steady-state response of them can be obtained by inverse fourier transform.

$$\{u_{ot}(t)\} = \frac{1}{2p} \int_{-\infty}^{\infty} \{\tilde{u}_{ot}(w)\} e^{iwt} dw \quad ; \quad \{\tilde{u}_{ot}(t)\} = \begin{Bmatrix} \tilde{u}_{oxt}(t) \\ \tilde{u}_{oyt}(t) \\ \tilde{g}_{ox}(t) \\ \tilde{g}_{oy}(t) \\ \tilde{q}_{o}(t) \end{Bmatrix} \tag{6}$$

Numerical Example

In order to illustrate the developed method of analysis and to assess the dynamic soil-structure interaction (SSI) effects on the three dimensional building with eccentricity in only one direction, as the configuration shown in Figure 1 was considered. The main parameters of the dynamical structure-foundation model and properties of the soil are summarized in Table 1. The mono-symmetric single-story building resting on a layer which is homogeneous and isotropic through a rigid square foundation consists of reinforced concrete frames joined at each floor level by a rigid diaphragm. The floor, including the foundation were assumed to be identical with length of the building d=12m and the width of the building b=12m. In this example, the center of stiffness is assumed to lie at eccentricity e=0.25m from center of mass. It has been assumed that floor slab and the foundation mat have the same eccentricity along the x axis in the direction perpendicular to the input motion, and no eccentricity along the y axis. The mass m=50t and torsional radius of gyration r=5m at floor level are taken into account. The rigid foundation mat is idealized as a circular plate of radius r_o=5m and its mass also taken to be m_o=50t. It should be noted that the radius of the base mass is taken as the radius of a circle having the same area as the plane of the structure. It is considered that the structure has the same translational stiffness in the x and y directions, and that the SSI translational and rocking stiffnesses are respectively the same in both horizontal directions. The stiffnesses K_{xB}, K_{xC} and K_{yB}=26500 kN/m of these frames and the height, h=3m of the story are considered. The viscoelastic foundation medium is assumed to have a density of 20 kN/m^3 and a Poisson's ratio of ν=0.33, and the material damping ratio is β=0.05. To indicate the significant of SSI effects on structural response, the shear wave velocity (c_s) of the layer material was selected as 500 m/s. The responses of this building-foundation system with the torsional effects have been established when subjected to Erzincan 1992 earthquake (E-W component, M=6.8) with a peak acceleration of 0.5g as the free field ground motion perpendicular to the direction of the eccentricity. The effects of the depth (D) of the embedment and the thickness (H) of the layer resting on a rigid substratum has been investigated to determine the behavior of the system during SSI in 3D.

Table 1. System parameters for numerical example

Model	Description	Symbol	Value
Structure	Floor mass	m	50 t
	Frame stiffness	K_{xB}, K_{xC}, K_{yB}	26500 kN/m
	Foundation to storey height	h	3m
	Floor moment of inertia	I	120 tm^2
	Elasticity modulus	E	28500000 kN/ m^2
Foundation	Mass	m_o	50 t
	Length & width	b, d	12m, 12m
	Moment of Inertia	I_o	120 tm^2
	Ground motion record	$\ddot{u}_{gy}(t)$	Erzincan 1992 (E-W)
Soil	Poisson's ratio	ν	0.33
	Mass density	ρ	20 kN/m^3
	Material damping	β	0.05
	Shear velocity	c_s	500 m/s

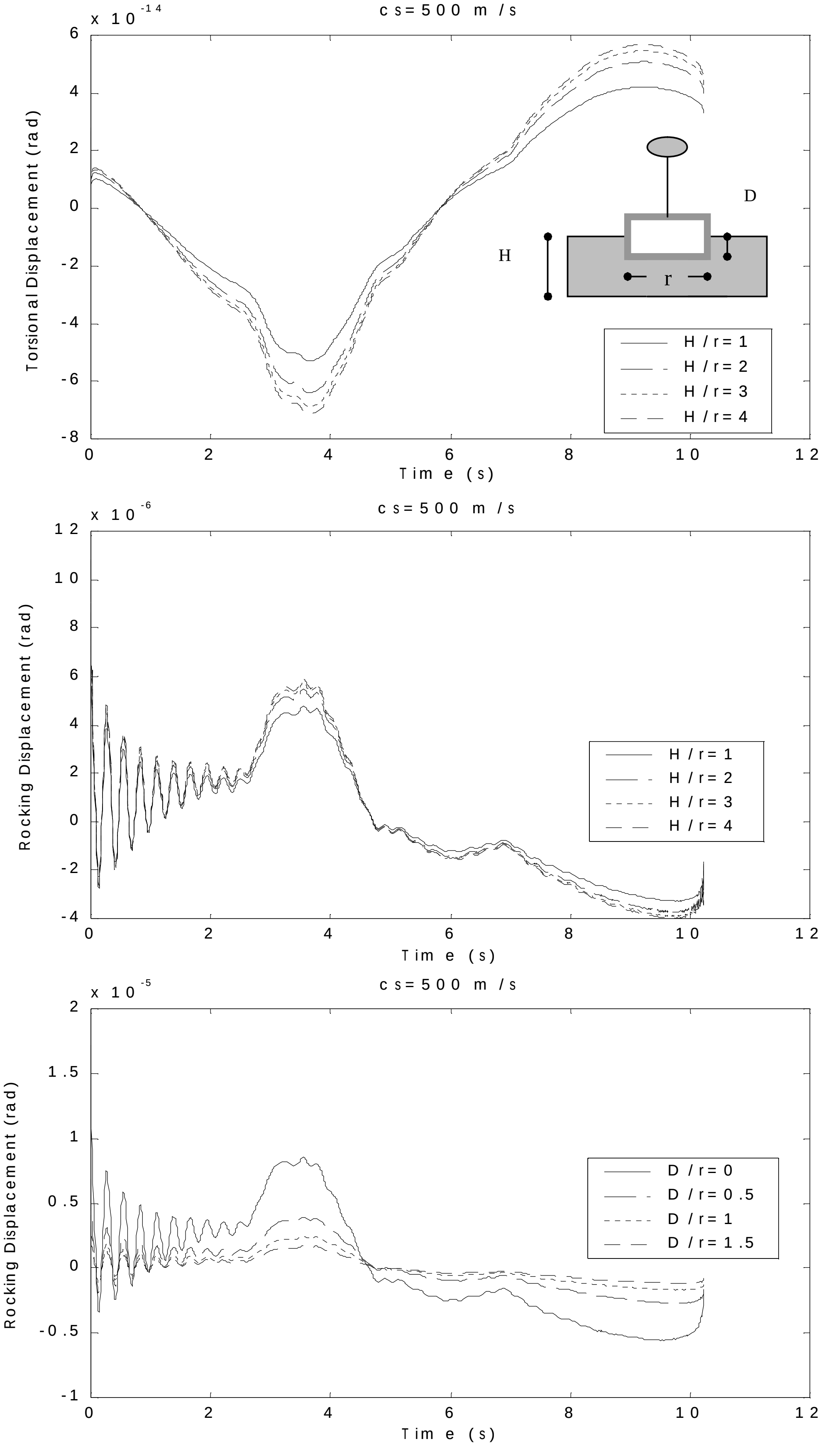

Figure 2. Variation of the displacement components of foundation versus time in the case of m/m_o=1 for intermediate eccentricity

The pratical range of variables considered in this study are taken as $1 \leq H/r \leq 4$ for the layer thickness ratio, $0 \leq D/r \leq 1.5$ for the embedment depth ratio. In Figure 2, the variation of rocking and torsional components of the foundation base for different layer thickness ratio(H/r) are plotted versus time.

The maximum values of the foundation displacements during Erzincan 1992 excitations are γ_o=4.75×10^{-6} rad and θ_o=5.25×10^{-14} rad for shear velocity of c_s=500m/s (intermediate soft soil condition) in the case of H/r=1 whereas the corresponding values are γ_o=6.25×10^{-6} rad and θ_o=7.75×10^{-14} rad in the case of H/r=4. Four different values of depth of embedment to gyration radius ratio (D/r) are considered. It should be noted that an apparent decreases occurs in the foundation rocking displacement relating the intermediate layer thickness case as embedment depth ratio increases in Figure 2.

Conclusions

A simplified methodology of analysis for the seismic response of three-dimensional single-story buildings idealized the whole structure-foundation-soil system with a lumped parameter model with frequency-dependent foundation impedance functions for the two cases of a site, surface and embedded footing-layer on rigid substratum is presented. From these time-domain responses, it has shown that an obvious increase occurs in torsional and rocking displacements of the foundation as the thickness of layer increase. When the depth of the embedment increases, the system response, especially the rocking displacement of the foundation decreases during building-soil interaction.

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