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# Inelastic Soil-Pile-Structure Interaction under Static Loading

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**ABSTRACT:** In this paper, inelastic pile-soil-structure interaction under static loading is presented by using finite element and boundary element methods. Linear beam column finite elements are used to model the piles and structural elements. Nonlinear soil is modeled by using nonlinear springs along the piles. Modified Özdemir's nonlinear model is implemented for springs and systems of equations are coupled for piles and pile groups at the interface nodes. By using this mixed type of numerical solution, it is possible to get a computationally efficient and accurate results. In order to verify the proposed formulation, the result of a full-scale load tests (performed by Rollins et al., 1996) are compared.

***Keywords :*** Pile-soil-structure interaction, boundary element method and finite element method, nonlinear analysis of piles.

**ÖZET:** Bu makalede, static yükler altında elastik olmayan zemin-kazık-yapı etkileşimi sonlu elemanlar ve sınır elemanlar metodlarını kullanarak sunulmaktadır. Kazık ve yapı elemanları için elastik kiriş-kolon sonlu elemanlar kullanılmaktadır. Lineer olmayan zemin, kazıklar boyunca lineer olmayan yaylarla modellenmektedir. Yaylar için; değiştirilmiş Özdemir'in modeli kullanılmakta ve kazık ile kazık grupları için denklem sistemleri etkileşim yüzeylerinde birleştirilmektedir. Bu tip karışık nümerik çözüm kullanarak, hesaplama açısından verimli ve doğru sonuçlar almak mümkün olabilmektedir. Önerilen formülasyonu doğrulamak için Rollins (1996) tarafından yapılan tam ölçekli yükleme testlerinin sonuçları karılaştırılmaktadır.

## Introduction

A number of methods of analyses for pile and pile groups had been developed in the past decade. The approaches can be classified into three categories: (i) The beam on Winkler formulation which simulates the soil resistance against the pile by a set of independent, linear and nonlinear distributed springs. (ii) The elastic continuum approach which involves the integration of Mindlin equations for displacements due to a surface point loading acting within an elastic halfspace. (iii) Finite element formulation, which discretizes both pile and surrounding soil onto finite elements, enforcing boundary conditions at the pile soil interface.

In the first category, load displacement curves of springs are assumed to be known a priori. These approach is also known as ‘p-y’ and ‘t-z’ methods (Coyle and Reese, 1966). The nonlinear curves used in analysis are empirical and based on observations of past experiment results. In the second method, the use of Mindlin’s solution was used in a linear boundary element formulation (Poulus,1968; Banerjee 1978). Some finite element method approaches were attempted by Desai (1974), Randolph (1981). Besides them, Pressley and Poulos (1986) used an elastic perfectly plastic soil model in an axisymmetric finite element method to approximately analyze pile groups.

It is essential to develop economically reliable solution technique for solution of pile-soil-structure interaction by taking into account the computational power of each method. This is done by hybrid approach that blends the approaches into one solution. The present study aims to develop this approach, which incorporates the effects of nonlinearities and material nonlinearity approximately into a well established continuum formulation. To verify the proposed algorithm, the result of a full-scale load tests (performed by Rollins et al., 1996) are compared.

## Continuum Formulation

### Pile Formulation

Pile is modeled as a linear two dimensional beams characterized by , $E_p$ Young’s modulus of pile material; $I_p$ second moment of inertia of pile; $A_p$ the cross-sectional area of pile, $u_x$ and $u_z$ lateral and axial displacements. The general solution to these equations rewritten in incremental matrix form :

$$\{\dot{u}_p\} = [D]\{\dot{t}_p\} + \lfloor b_p \rfloor \{\dot{u}_c\} \tag{1}$$

where, $[D]$ is a coefficient matrix, $\lfloor b_p \rfloor$ is a boundary condition matrix for a unit pile head displacements and rotations, $\{\dot{u}_c\}$ is the vector of pile head displacements and rotations, $\{\dot{t}_p\}$ is the incremental tractions, and the subscript ‘*p*’ represents the displacements and tractions obtained by pile domain only.

### Soil Model

The static Green’s functions are used in solving the governing differential equation of a semi-infinite elastic half-space. This is well established by Banerjee (1978) and used extensively to study the elastic behavior of axially and laterally loaded pile groups. By discretizing the pile soil interface into cylindrical elements, and integrating along the surface, the matrix form of soil equations in an incremental form can be represented as:

$$[G]\{\dot{t}_s\} = \{\dot{u}_s\} \tag{2}$$

where $\{\dot{u}_s\}$ is the incremental displacements of the soil; $[G]$ is the Green's function for the half space, $\{\dot{t}_s\}$ is the incremental tractions at the pile soil interface.

**Nonlinear Model**

A one-dimensional simple method was developed by Özdemir (1976). Due to its simplicity and requiring only a few parameters, it is very attractive to use it. Besides its simplicity, it provides hysteretic rule that is needed to model the cyclic behavior (Kucukarslan, 1999). This model is used to relate displacements and tractions. The modified Özdemir's model is written as:

$$\dot{t} = K\left[\dot{w} - |\,\dot{w}\,|\left(\frac{t-b}{Y}\right)\right] \tag{3}$$

where

$t =$ traction and $\dot{t} =$ traction rates ( increment)
$K =$ elastic modulus
$\dot{w} =$ displacement rates (increment)
$b =$ back stress
$Y =$ yield stress

**Coupling of Pile and Soil Equations**

By imposing a constraint on the traction vector $\{\dot{t}_p\}$, the algebraic equations for soil domain and pile domain can be coupled as following form:

$$[B]\{\dot{t}_p\} = \{\dot{f}_c\} \tag{4}$$

where, $[B]$ is matrix dependent on pile geometry, material properties of pile and soil; $\{\dot{f}_c\}$ is the vector of externally applied loads on pile cap.

**Assembly of Pile Soil Interface**

Pile and soil to be related by:

$$[K_{ozd}](\{\dot{u}_s\} - \{\dot{u}_p\}) = \{\dot{t}_p\} \tag{5}$$

where $K_{ozd}$ is a nonlinear artificial stiffness which depends on the history of displacements. The incremental traction vectors $\{\dot{t}_p\}, \{\dot{t}_s\}$ and incremental displacement vectors $\{\dot{u}_p\}, \{\dot{u}_s\}, \{\dot{u}_c\}$ are unknowns. In order to couple these above equations, displacement compatibility relations have to be ensured.

Equilibrium at pile soil interface is satisfied by:

$$\{\dot{t}_s\} = -\{\dot{t}_p\} \tag{6}$$

Using equations (5) and (6 )and vanishing $\{u_s\}$from equations (1) and (3) results in:

$$\begin{bmatrix} G+D+K_{ozd}{}^{-1} & b_p \\ B & 0 \end{bmatrix} \begin{Bmatrix} t_p \\ u_c \end{Bmatrix} = \begin{Bmatrix} 0 \\ f_c \end{Bmatrix} \tag{7}$$

where $K_{ozd}$ is the equivalent nonlinear stiffness given by Özdemir's model and can be represented as:

$$K_{ozd} = K\left[1-\operatorname{sgn}(w)\left(\frac{t-b}{Y}\right)\right] \tag{8}$$

**Superstructure Equations**

A generalized finite element formulation for a structure will be constructed. Detailed discussion can be referred to Clough and Penzien (1993). After the equations for the entire super structure are established, a sub-structure method is used to couple the system to the foundation system.

$$\begin{bmatrix} K_{ss} & K_{sg} \\ K_{gs} & K_{gg} \end{bmatrix} \begin{Bmatrix} u^t \\ u_g \end{Bmatrix} \begin{Bmatrix} f_t \\ 0 \end{Bmatrix} \tag{9}$$

where $\{u^t\}$ and $\{u_g\}$are the total displacements vectors corresponding to the superstructure and support degrees of freedom respectively.

By reducing the equation on $u_c$, one can rewrite eqn. (9) as:

$$[\overline{K}]\{u_c\} = \{f_c\} \tag{10}$$

where $[\overline{K}]$ is the effective complex stiffness matrix, and $\{f_c\}$ is the effective load vector.

**Assembly of Entire System**

The effect of super structure stiffness and inertial loading on the foundation system go directly into the global equilibrium equations. The final coupled equation takes the form:

$$\begin{bmatrix} G+D+K_{ozd}{}^{-1} & b_p \\ B & \overline{K} \end{bmatrix} \begin{Bmatrix} t_p \\ u_c \end{Bmatrix} = \begin{Bmatrix} 0 \\ f_c \end{Bmatrix} \tag{11}$$

The solution of above equation gives the tractions along the pile-soil interface and the foundation displacement.

## Verification of Proposed Numerical Method

A series of static load tests were performed on a single pile and on a 3x3 pile group in soft clay by Rollins et al. (1996). These tests were on full scale and well instrumented pile and pile groups under lateral loads. A plan view of the test site is shown in figure 1. More detail can be found in Rollins et al. (1996).

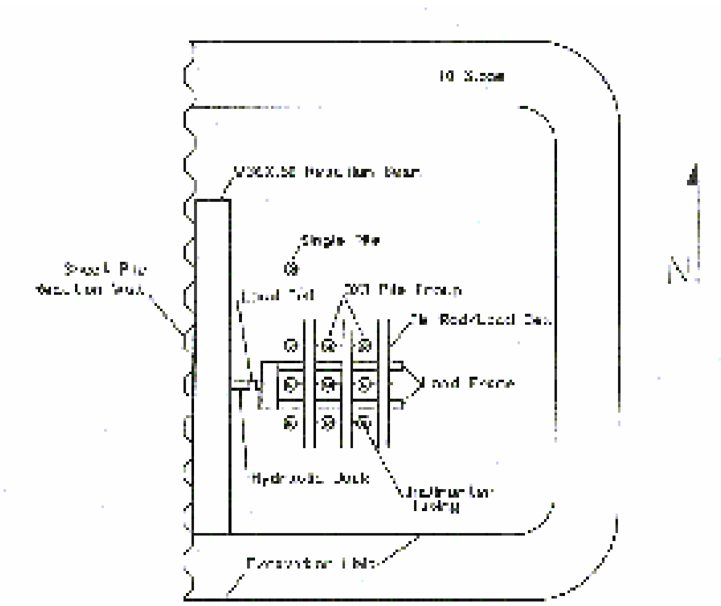

Figure 1. Plan view of pile group and structure.

### Single pile test

The single pile analysis is undertaken, because the analysis would make it possible to evaluate and compare the ability of computational method to match the measured response of the pile group using the same input parameters. These fitted parameters can be used in theoretical calculation for groups.

Computed results were obtained using material properties:

| Soil Modulus, $E_s$ | Shear Strength of soil, $c_u$ | Young's Modulus of pile, $E_p$ | Poisson's Ratio of soil, $u_s$ |
|---|---|---|---|
| 10.4 MPa | 50 kPa | 200 GPa | 0.4 |

Table 7.1 Parameters used in all analyses

Load versus pile head displacement curve for single pile is shown in figure 2. Predicted load displacements agree quite well. To assess the ability of computational method, depth versus maximum moment in the pile at specific loading is plotted in figure 3.

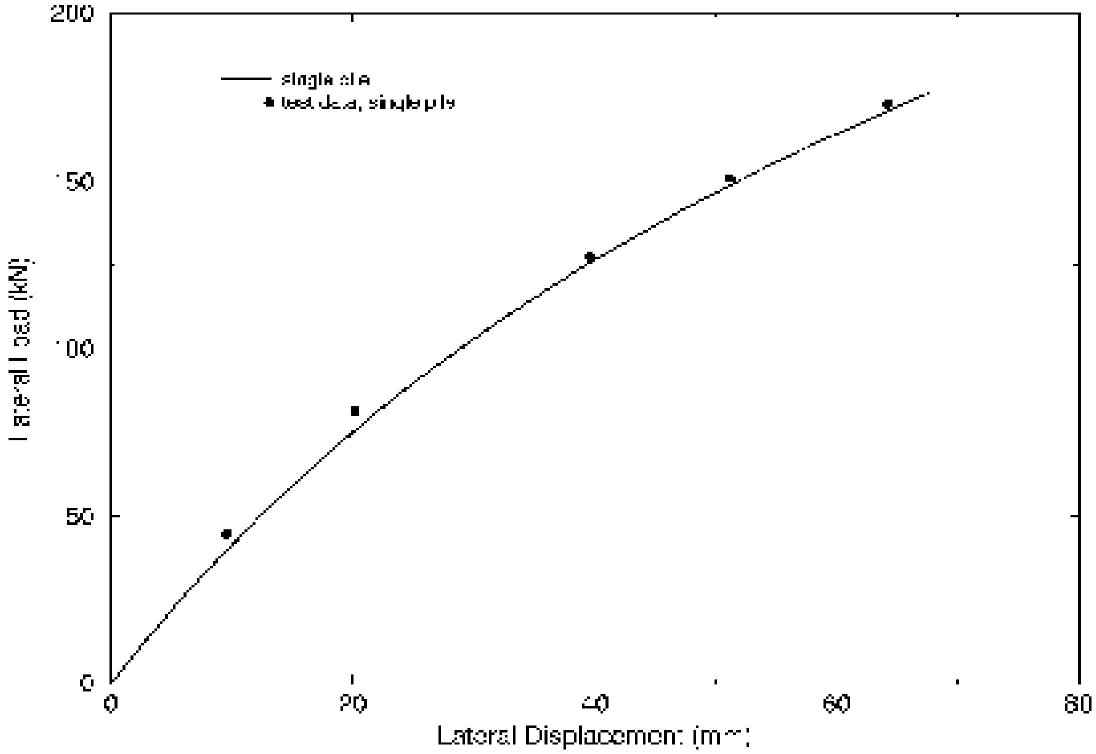

Figure 2. Lateral load vs. lateral displacement curve for single pile

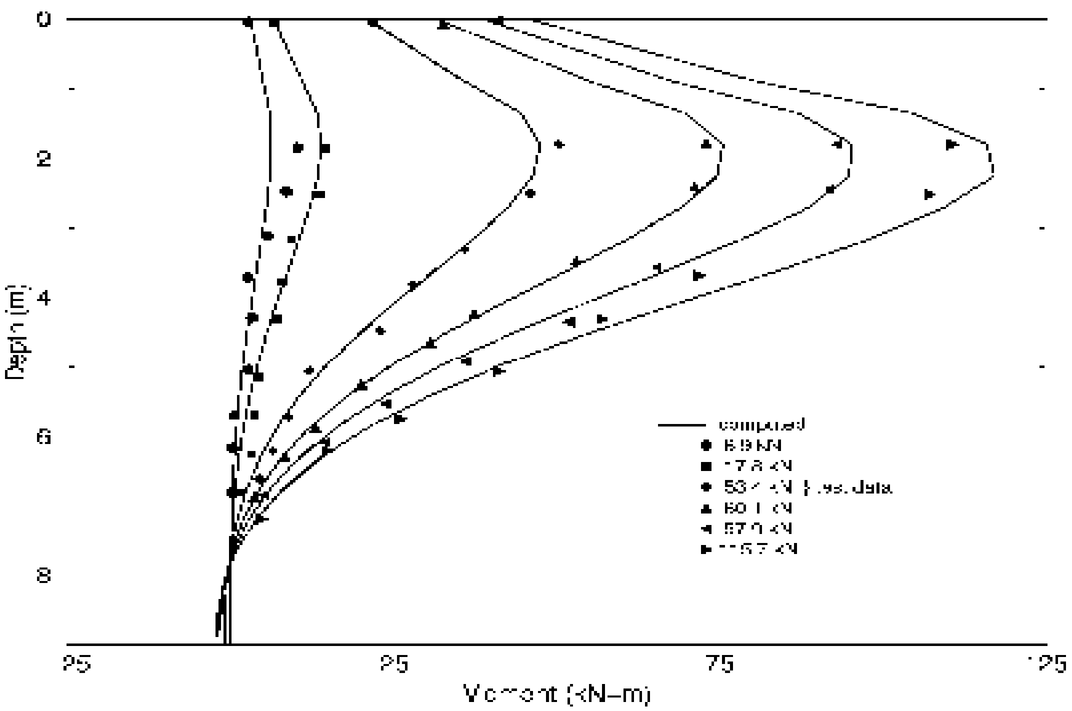

Figure 3. Depth vs. moment along the single pile

**Free head 3x3 pile group**

The main purpose of conducting the static, free head, pile group lateral load test was to determine the complex pile-soil-structure interaction of a pile group in soft clay. The results from the static group test were compared and normalized by the results from the single pile test results.

Pile head displacement versus load curve is plotted in figure 4. From the results, it is evident that each row carries some proportion of load. In experimental results, it was seen that front row carries the most of the load, and middle row carried the least amount. Besides load versus displacement curves moment distribution along the pile is also computed in figure 5. The magnitudes of moments and locations are similar and show similar trends.

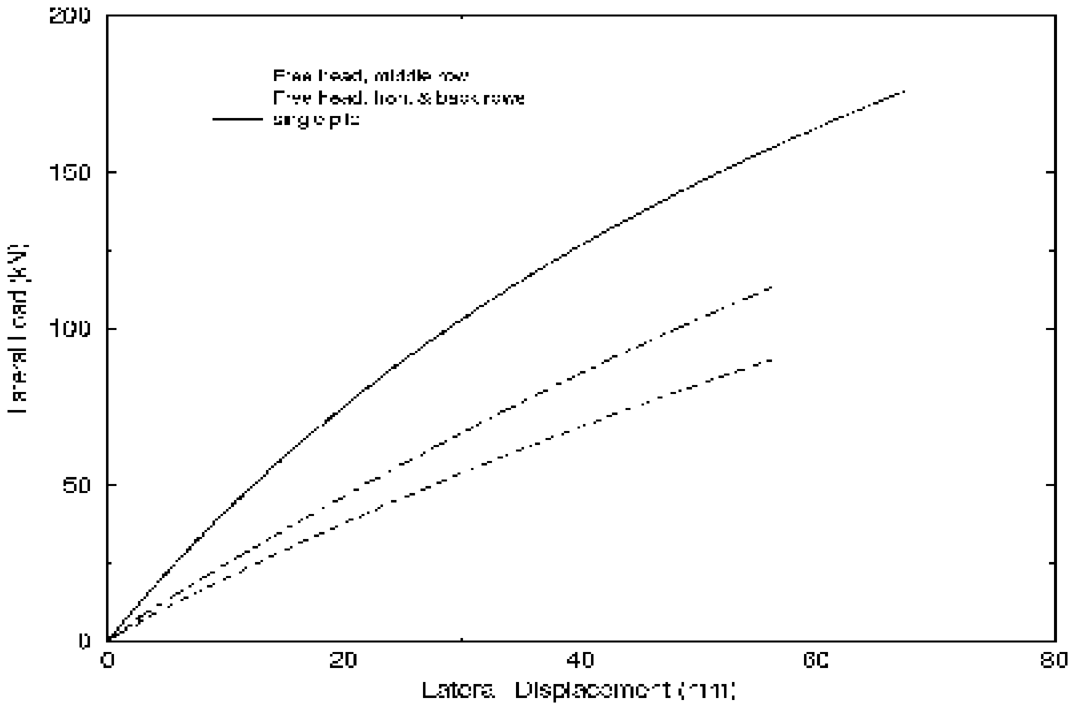


Figure 4. Row behavior of the group

## Conclusions

Pile soil structure interaction under static loads was formulated by using most efficient numerical methods, BEM and FEM. Nonlinear behavior was accommodated by hysteretic springs having a constitutive relation of modified Ozdemir’s model. To verify the proposed method, a numerical study was done and compared with available test data. It was seen that results are reasonably agrees with those of experiment results. The key to the successful prediction of piles/pile groups settlements are not only the method used but also the proper assessment of the soil parameters and in particular, the magnitude and the distribution of Young’s modulus of the soil.

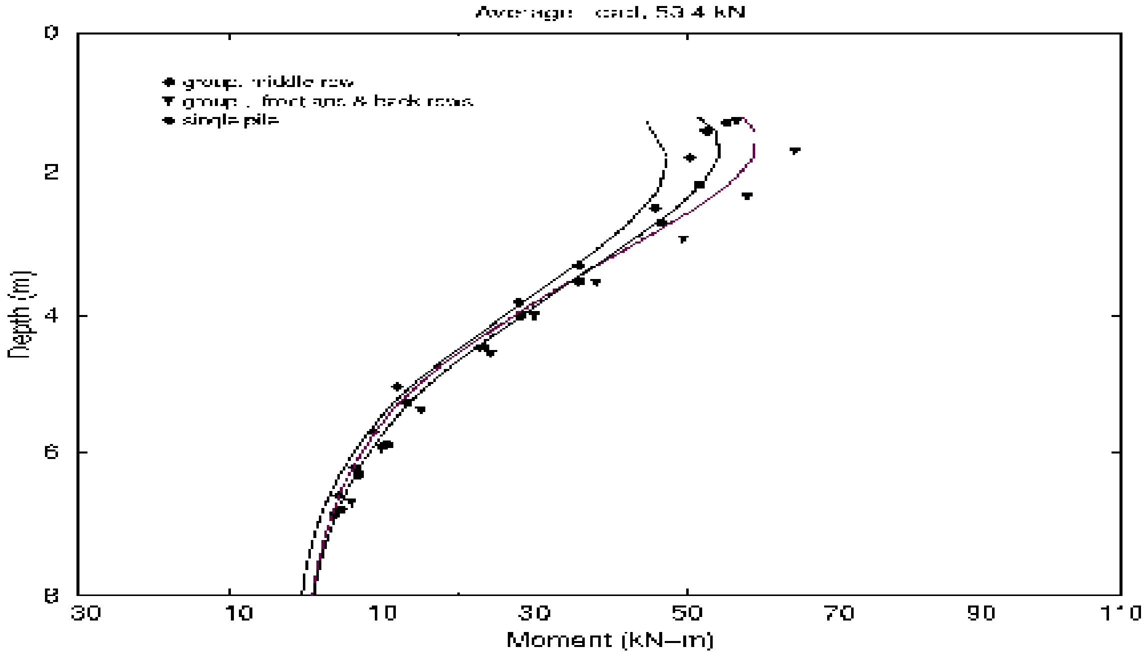


Figure 5. Moment Distribution

## References


Banerjee, P.K.,1978, Analysis of axially and laterally loaded pile groups, *Developments in Soil Mechanics,* Applied Science Publishers, London, pp. 317-346.

Clough, R.W., Penzien, J., 1993, *Dynamics of Structures* , McGraw-Hill, Inc.

Coyle, H.M., Reese, L.C., 1966, Load transfer for axially loaded piles in clay, *J. Soil Mech. Found. Engng. Div.,* Vol. 92, No. 2, pp. 1-26.

Desai, C.S., 1974, Numerical design and analysis for piles in sand, *J. Geotch. Engng. Div., ASCE,* Vol. 100, No. 6, pp. 613-635.

Küçükarslan, S., Linear and nonlinear soil-pile-structure interaction under static and transient impact loading, *PhD dissertation*, State University of New York, 1999.

Özdemir, H., Nonlinear transient dynamic analysis of yielding structures, *PhD dissertation,* University of California. Berkeley, CA, 1976.

Poulos, H.C.,1968, Analysis of the settlement of pile group, *Geotechnique,* Vol. 18, pp. 449-471.

Pressley, J.S., Poulus, H.G., 1986, Finite element analysis of mechanisms of pile group, *Int. J. Numer. Anal. Methods Geomech.*, Vol. 10, pp. 213-221.

Randolph, M.F.,1981, The response of flexible piles to lateral loading, *Geotechnique,* Vol. 31, No. 2, pp. 247-259.

Rolllins, K., Peterson, K., Weaver, T., Full-scale pile group lateral load testing in soft clay, *Technical Report,* Nat. Cent Earthq. Engng Res., Brigham Young Univ., 1996.