Free Vibration Analysis of Multi-bay Coupled Shear Walls

O. Aksogan, M. Bikce and R. Resatoglu

Civil Engineering Department, Cukurova University, Adana 01330, Turkey

ABSTRACT: The present study considers the free vibration analysis of stiffened multibay coupled shear walls on elastic foundation. The connecting beam properties and the thickness of the wall may be varied in a special manner. Continuous connection method (CCM) is employed to find the system stiffness matrix. The system mass matrix is found with the lumped mass assumption. A computer program has been prepared in MATHEMATICA computer algebra system and some sample problems have been solved. The results obtained using the foregoing program have been compared with those obtained using SAP2000 structural analysis program, ending up with a good match.

Keywords: multi-bay coupled shear wall; stiffening beam; continuous connection method; free vibration analysis; elastic foundation, natural frequency

ÖZET: Bu çalışmada, elastik temel üzerine oturan, sonlu sayıda güçlendirici kirişi olan, bağlantı kirişi özellikleri boşluktan boşluğa ve/veya bölgeden bölgeye değişebilen çok sıra boşluklu perdelerin serbest titreşim analizi yapılmıştır. Sistem rijitlik matrisi oluşturulurken sürekli bağlantı yöntemi (SBY) kullanılmıştır. Sistem kütle matrisi, toplanmış kütle kabulüne göre yükseklik boyunca istenilen sayıda toplanmış kütle alınarak elde edilmiştir. Çalışmanın sonunda, MATHEMATICA programlama dilinde bir program hazırlanarak sunulan yöntemle bazı örnekler çözülmüştür. Elde edilen sonuçlar SAP2000 yapı analizi programı ile elde edilen sonuçlarla karşılaştırılarak sonuçların çok yakın olduğu gözlenmiştir.

Introduction

In tall buildings, wind and earthquake induced lateral forces are generally resisted by shear walls. A solid shear wall can easily be accounted for as a cantilever beam. Whereas, shear walls pierced by doors, windows and corridors are harder to analyze since they are highly indeterminate. Due to their high resistance to lateral forces, shear walls have become very popular in tall buildings. Thus, an increase in the number of bays in such structures for architectural needs has become inevitable.

During design, it is important to know the free vibration properties of a structure, to carry out its computation for dynamic lateral loading. For this purpose, Li and Choo (1984) have studied the free vibration analysis of a single bay coupled shear wall and

determined the natural frequencies and the corresponding mode shapes. In the present study, a free vibration analysis has been carried out for a multi-bay coupled shear wall on elastic foundation.

If the CCM is applied on multi-bay coupled shear walls directly, the free vibration analysis carried out ends up with a high order system of simultaneous differential equations, written for all sections of the shear wall. Hence, a suitable special application has been employed to render the problem manageable.

The special method used in the present study comprises two steps. In the first step, the structure is modeled as a system of lumped masses. The number of lumped masses gives the degrees of freedom of the system and is selected freely by the analyst. Each lumped mass is determined by using the average mass per unit height of the corresponding section of the shear wall.

To find the stiffness matrix, each lumped mass is loaded with a unit horizontal force at a time and the corresponding horizontal displacement vectors for the whole structure is found. For each loading, the compatibility equation is written for the vertical displacements at the midpoints of the connecting beams in each span using the CCM. To solve the system of second order, linear, coupled differential equations, thus obtained, first, a matrix orthogonalization is applied to uncouple it (Meirovitch, 1980). Then, this system of equations with diagonal coefficient matricies is solved numerically. During this analysis, the boundary conditions at the base of the shear wall are written, accounting for the vertical, horizontal and rotational stiffnesses of the foundation. During the determination of the horizontal displacements, the horizontal displacements and slopes of any two neighboring sections and the stiffener (if there is one) at that boundary are taken to be equal.

After finding the displacements for all unit loading cases, the flexibility matrix for the multi-bay shear wall can be written in a straightforward manner. The inverse of this matrix is the stiffness matrix. Substituting this stiffness matrix and the previously obtained mass matrix in the free vibration equation, the natural frequencies and the corresponding mode vectors are obtained.

A computer program has been prepared in the MATHEMATICA computer algebra system to implement the foregoing analysis. At first, the results of the present work for a single bay coupled shear wall has been compared with those given by Li and Choo (1984). Then, the free vibration analysis of a five bay coupled shear wall, has been carried out. Both examples were solved both by the present method and the structural analysis program SAP2000 (Wilson, 1997) and the results were observed to be in very good agreement.

Analysis

The multi-bay coupled shear wall is, first, modeled by discrete masses, and then, analyzed by the CCM. The mass matrix of the multi-bay coupled shear wall is found as a diagonal matrix employing a lumped mass approach. For this purpose, the top, the bottom and each height at which there is a stiffening beam and/or change of wall properties will be called "ends" and the region between any two consecutive ends will be called a "section". A suitable number of equidistant masses will be placed in each section. The masses in a section will be found by dividing the total mass of the section by this number and half of that will be assigned to the ends of the section. Completing this procedure for each section and adding to each end the additional mass due to the

pertinent stiffening beam, the stiffness matrix is found as a diagonal matrix. The dimension of this matrix is NxN where N shows the number of masses (Figure 1). In figure 1, i shows the number of sections and borderlines between consecutive sections, j shows the number of bays and piers, m shows the total number of bays and n shows the total number of sections in vertical direction.



Figure 1. A multi-bay shear wall with stiffening beams

Ignoring the vertical and rotational inertial effects, only the horizontal one is taken into consideration. Thus, each lumped mass contributes only one degree of freedom. Although this simplification causes some error in the higher modes, an increase in the number of lumped masses decreases this error. Despite the foregoing simplification, a more important factor, i.e. the flexibility of the foundation, is accounted for in the present study. Towards this end, the bottom end of each pier has been modeled by three constraining springs, one vertical, one horizontal and the third rotational.

The primary assumption in the CCM is that the lengths of the connecting beams do not change, i.e. their axial stiffness is infinitely large. This assumption is equivalent to the widely used rigid diaphragm model for storey floors which is known to yield rather good results. This assumption renders the lateral displacements at the same floor level equal, for all piers. Consequently, the slopes and curvatures at the same level can be assumed to be equal, as well. Furthermore, in this method, it being assumed that every span between two neighboring piers is of constant value throughout the total height of the wall, the real connecting beams with bending stiffness $EI_{cj,i}$ are replaced by a laminated medium with $EI_{cj,i}/h_i$ per unit length in the vertical direction. In the foregoing expression, E, $I_{cj,i}$ and h_i are, respectively, the elasticity modulus, the moment of inertia of the connecting beams in bay j of section i and the storey height in section i. Likewise, the discrete shear forces in the connecting beams are replaced by a

continuous shear force function $q_{j,i}$, per unit length of height, along the mid-points of the connecting laminae (i.e. the points of zero moment).

The flexibility matrix is found by applying a unit force in the horizontal direction at the height of each lumped mass, one at a time. The horizontal displacements found from each unit loading case constitute a column of the flexibility matrix. Hence, an analysis carried out for one general unit loading will suffice to find the complete flexibility matrix, the inverse of which yields the stiffness matrix.

Now that the discrete values are expressed as a continuous function of the longitudinal coordinate, to find the relations among the shear force functions, $q_{j-1,i}$ and $q_{j,i}$, and the corresponding contributions to the axial forces in the piers, $Q_{j-1,i}$ and $Q_{j,i}$, the connecting laminae, on the two sides of a pier in section i, are cut at their midpoints, which are the points of zero moment (Figure 2).



Figure 2. The vertical forces on an isolated region of the shear wall

Applying the vertical force equilibrium equation to a dx length of pier j of section i, the following equations are found:

$$\frac{dQ_{j-1,i}}{dx} - \frac{dQ_{j,i}}{dx} = q_{j,i} - q_{j-1,i} \quad j = 1, 2, \dots, m+1, \quad i = 1, 2, \dots, n$$
(1)

in which

$$Q_{0,i} = Q_{m+1,i} = q_{0,i} = q_{m+1,i} = 0$$
 $j = 1,2,...,m, i = 1,2,...,n$ (2)

It should be noted, here, that instead of the axial force in section i of pier j, the sum of the shear forces starting from the top in the two neighboring spans, namely $Q_{j,i}$ and $Q_{j-1,i}$ functions, are taken as the fundamental unknowns. The difference between the foregoing functions constitutes the axial force in section i of pier j. As a result of equations (1-2)

$$\frac{dQ_{j,i}}{dx} = -q_{j,i} \quad j = 1, 2, ..., m + 1, \quad i = 1, 2, ..., n$$
(3)

When the unit force is in one of the sections, defined previously, that section is divided into two new sections.

Defining the Macaulay's brackets by

$$< x - x' >^{n} = (x - x')^{n} \text{ and } < x - x' >^{0} = 1 \text{ for } x > x' \\ < x - x' >^{n} = 0 \text{ and } < x - x' >^{0} = 0 \text{ for } x \le x' \end{cases}$$
(4)

the moment-curvature relation, in a generalized sense, for a cross-section of the shear wall at height x, can be written as follows:

$$EI_{i} \frac{d^{2}y_{i}}{dx^{2}} = \langle H_{p} - x \rangle^{1} - \sum_{j=1}^{m} Q_{j,i}L_{j} \quad i = 1, 2, ..., n$$
(5)

Here, H_p , L_j and I_i are, respectively, the height at which the unit force is applied, the distance between the axes of the piers j and j+1 and the sum of the moments of inertia of the piers in section i.

It will be assumed that all rows of connecting laminae will be cut through the midpoints, which are the points of zero moment, thus exposing the shear forces in them. The compatibility of the relative vertical displacements at the ends, on the two sides of the cut sections necessitates their sums to be equal to zero for each lamina, i.e.

$$L_{j} \frac{dy_{i}}{dx} - \frac{h_{i}a_{j}^{2}}{2C_{cbi}}q_{j,i} - \frac{h_{i}a_{j}^{3}}{12EI_{cj,i}}q_{j,i}$$

$$- \frac{1}{E} \sum_{k=i+1}^{n} \left[\left\{ \frac{1}{A_{j,k}} \right\}_{x_{k+1}}^{x_{k}} (Q_{j,k} - Q_{j-1,k}) dx + \left\{ \frac{1}{A_{j+1,k}} \right\}_{x_{k+1}}^{x_{k}} (Q_{j,k} - Q_{j+1,k}) dx \right]$$

$$- \frac{1}{E} \left[\left\{ \frac{1}{A_{j,i}} \right\}_{x_{i+1}}^{x} (Q_{j,i} - Q_{j-1,i}) dx + \left\{ \frac{1}{A_{j+1,i}} \right\}_{x_{i+1}}^{x} (Q_{j,i} - Q_{j+1,i}) dx \right] - \delta_{0} = 0$$

$$j = 1, 2, ..., m, \quad i = 1, 2, ..., n \qquad (6)$$

This compatibility equation can be written for all spans of the shear wall, having in mind the previously given values (2). In (6), a_j , C_{cbi} , $I_{cj,i}$, $A_{j,i}$ and δ_{0j} are, respectively, the open length of span j, the beam-wall connection stiffness in section i, the moment of inertia of the connecting beams in span j of section i, the cross-sectional area of pier j in section i and the relative vertical displacement of the bottom of pier j with respect to that of pier j+1. The terms of the compatibility equation (6) are the relative vertical displacements of the two ends on the two sides of the cut due, respectively, to the bending of the piers, the relative rotation of the beams with respect to the piers of the piers between section i and the foundation, the axial deformations of the piers in section i and the relative vertical displacements in the foundation. Differentiating each and every equation in (6) with respect to x, substituting expressions (3) and (5) and simplifying the resulting equations, the following non-homogeneous second order linear differential equation with constant coefficients is obtained for each section i (i=1,2,...,n):

$$\left[\gamma_{j,i}^{2} Q_{j,i}''\right]_{mx1} - \left[\alpha_{jk}^{2}\right]_{mxm} \left[Q_{j,i}L_{j}\right]_{mx1} = -\langle H_{p} - x \rangle^{1} \quad j = 1, 2, ..., m, \quad x_{i+1} \leq x \leq x_{i}$$
(7)

where the following definitions apply:

$$\begin{aligned} \alpha_{jk}^{2} &= \left(1 + \frac{I_{i}}{L_{j}^{2}} \left(\frac{1}{A_{j,i}} + \frac{1}{A_{j+1,i}} \right) \right) \quad (j = k) \\ \alpha_{jk}^{2} &= \left(1 - \frac{I_{i}}{L_{j}L_{j-1}A_{j,i}} \right) \quad (j = k+1) \\ \alpha_{jk}^{2} &= \left(1 - \frac{I_{i}}{L_{j}L_{j+1}A_{j+1,i}} \right) \quad (j = k-1) \\ \alpha_{jk}^{2} &= 1 (j < k-1) \\ \alpha_{jk}^{2} &= 1 (j > k+1) \\ \gamma_{j,i}^{2} &= \frac{EI_{i}}{L_{j}} \left(\frac{h_{i}a_{j}^{2}}{2C_{cbi}} + \frac{h_{i}a_{j}^{3}}{12EI_{cj,i}} \right) \quad k = 1, 2, ..., m, \quad j = 1, 2, ..., m, \quad i = 1, 2, ..., n \end{aligned}$$

$$(8)$$

The set of differential equations (7) is coupled and, moreover, as the number of bays increase, closed form solution is not feasible, if at all possible. In this work, matrix orthogonalization method will be used for solving the differential equation set (7). For this purpose, first, using the variable transformation

$$Q_{j,i}L_{j} = Z_{j,i}$$
 $j = 1, 2, ..., m, i = 1, 2, ..., n$ (9)

the equation set (7) can be written in the following form for the new variables $Z_{j,i}$

$$\begin{bmatrix} \frac{\gamma_{1,i}^{2}}{L_{1}} & 0 & \cdot & 0 \\ 0 & \frac{\gamma_{2,i}^{2}}{L_{2}} & & 0 \\ \cdot & \ddots & \cdot & \cdot \\ 0 & 0 & \cdot & \frac{\gamma_{m,i}^{2}}{L_{1}} \\ \frac{1444444244444}{\underline{M}2444444} \end{bmatrix} \begin{bmatrix} Z_{1,i}^{"} \\ Z_{2,i}^{"} \\ \vdots \\ \frac{Z_{2,i}^{"}}{L_{2}} \\ \frac{Z_{2$$

where <u>A</u> and <u>B</u> are mxm and <u>Z</u> and <u>M_e</u> are mx1 dimensional matrices. The homogeneous part of this matrix equation, which is an eigenvalue problem in the following form

$$\underline{\mathbf{A}} \quad \underline{\mathbf{Z}''} + \underline{\mathbf{B}} \quad \underline{\mathbf{Z}} = \underline{\mathbf{0}} \tag{11}$$

is solved and the eigenvectors corresponding to the eigenvalues all together yield the transformation matrix T. Since the coefficient matrices <u>A</u> and <u>B</u> are constant, (10) can be diagonalized. For this purpose, the following transformation can be used:

$$\underline{Z} = \underline{T} \ \underline{Y}, \qquad \underline{Z''} = \underline{T} \ \underline{Y''}$$
(12)

which, when substituted in (10), yield

$$\underline{\mathbf{A}} \quad \underline{\mathbf{T}} \quad \underline{\mathbf{Y}''} + \underline{\mathbf{B}} \quad \underline{\mathbf{T}} \quad \underline{\mathbf{Y}} = \mathbf{M}_{e} \tag{13}$$

Here, \underline{T} and \underline{Y} are, respectively, the transformation matrix of dimension mxm and the vector of independent functions of variable x of dimension mx1. Multiplying both sides of (13) by the transpose of \underline{T} , the two coefficient matrices \underline{A} and \underline{B} are diagonalized to yield

$$\underline{\tilde{A}} \quad \underline{Y''} + \underline{\tilde{B}} \quad \underline{Y} = \underline{\tilde{M}}_{e} \tag{14}$$

which is an uncoupled system of differential equations. The set of expressions yielding the solution of (14) for all sections is of the following form:

$$Y_{j,i} = C_{j,i} \operatorname{Cosh}\left(\sqrt{\frac{\widetilde{B}_{jj,i}}{\widetilde{A}_{jj,i}}}x\right) + D_{j,i} \operatorname{Sinh}\left(\sqrt{\frac{\widetilde{B}_{jj,i}}{\widetilde{A}_{jj,i}}}x\right) + \frac{1}{\widetilde{B}_{jj,i}} < H_p - x >^1$$

$$j = 1, 2, ..., m, \quad i = 1, 2, ..., n$$
(15)

There are 2xmxn integration constants $C_{j,i}$ and $D_{j,i}$ in (15). To determine these constants, the boundary conditions at the top, bottom and between each pair of consecutive sections are used.

Before writing down the boundary conditions, the shear forces in the stiffening beams must be determined. For this purpose, compatibility equation (6) must be written both for section i at level x_i and the stiffening beam i and solved simultaneously. Thus, employing the definitions

$$\eta_{j,i} = \frac{1}{H} \frac{\left(\frac{h_i}{C_{cbi}} + \frac{h_i a_j}{6EI_{cj,i}}\right)}{\left(\frac{1}{C_{sbi}} + \frac{a_j}{6EI_{si}}\right)} \quad j = 1, 2, ..., m, \quad i = 1, 2, ..., n$$
(16)

the shear forces in the stiffening beams are found as follows:

$$V_{j,i} = -\eta_{j,i} H \frac{dQ_{j,i}}{dx} \bigg|_{x=x_i} \qquad j = 1, 2, ..., m, \quad i = 1, 2, ..., n$$
(17)

Since there are as many unknown integration constant pairs, $C_{j,i}$ and $D_{j,i}$, as there are spans, the conditions of vertical force equilibrium at the ends including the cross-section where the unit force acts and excluding the bottom of the wall and the conditions of continuity of slope at the ends including the cross-section where the unit force acts and excluding the cross-section where the unit force acts and excluding the cross-section where the unit force acts and excluding the top of the wall suffice to find them. While these conditions are written, the value of the axial force at the top of each pier is taken to be zero and the slope and the relative vertical displacement at the bottom of j'th pier must be taken, respectively, as follows:

$$\frac{dy_{n}}{dx}\Big|_{x=0} = \frac{H_{p} - \sum_{j=1}^{m} Q_{j,n} L_{j}}{\sum_{i=1}^{m+1} K_{ij}}$$
(18)

$$\delta_{0} = \frac{Q_{j,n} - Q_{j-1,n}}{K_{vj}} + \frac{Q_{j,n} - Q_{j+1,n}}{K_{v(j+1)}}$$
(19)

Here, K_{rj} and K_{vj} are, respectively, the rotational and vertical stiffnesses of the foundation of the j'th pier. When there is a stiffening beam at the bottom of the wall, $Q_{j,i}$ must be increased as much as the shear force in that stiffening beam.

Substituting the integration constants, obtained from the foregoing boundary conditions, in expressions (15), then, the resulting $Y_{j,i}$ in the first of equations (12) and the latter expressions in the second of equations (9), the unknown functions $Q_{j,i}$ are obtained, in a straightforward manner.

Substituting $Q_{j,i}$, thus obtained, in the moment-curvature relations (5) and integrating twice with respect to x, the following expressions are found for the horizontal displacements:

$$y_{i} = \frac{1}{EI_{i}} \int \left[\int \left(^{1} - \sum_{j=1}^{m} Q_{j,i} L_{j} \right) dx \right] dx + H_{i} x + G_{i} \quad i = 1, 2, ..., n$$
(20)

The number of integration constant pairs, H_i and G_i , in the above equations is equal to the number of sections along the height of the wall. These constants are determined from the continuity of the displacements and slopes at the ends between all pairs of consecutive sections and the conditions, at the bottom of the wall, that the horizontal displacement and the rotation at the bottom of the wall should be given, respectively, as follows:

$$y_{n}\Big|_{x=0} = \frac{1}{\sum_{j=1}^{m+1} K_{hj}}$$
 (21)

$$\frac{dy_{n}}{dx}\Big|_{x=0} = \frac{H_{p} - \sum_{j=1}^{m} Q_{j,n}L_{j}}{\sum_{j=1}^{m+1} K_{rj}}$$
(22)

Here, K_{hj} is the horizontal stiffness of the foundation of the j'th pier.

Having determined the lateral displacements for unit loadings at each and every one of the levels of lumped masses, the flexibility matrix and thereof the stiffness matrix can be obtained in a straightforward manner. Then, writing down the standard frequency equation for the discrete system in the following form:

$$\underline{\mathbf{M}} \ \underline{\mathbf{X}} + \underline{\mathbf{K}} \ \underline{\mathbf{X}} = \underline{\mathbf{0}} \tag{23}$$

solving it for the circular frequencies and substituting them back into the free vibration equation, one at a time, the corresponding mode shapes can be obtained.

Numerical Results

A computer program has been prepared using MATHEMATICA computer algebra system to implement the analysis carried out in the previous section. To verify the present method two examples have been solved.

Example 1 treats the natural vibration analysis of the single bay coupled shear wall in the literature (Li and Choo, 1984). This shear wall rests on rigid foundation and has no stiffeners, 0.3048 m thickness and the following properties: $a_1=2.438$ m, $b_1=b_2=6.096$ m, H=60.96 m, h=3.048 m, A_c=0.2127 m², I_c=8.63×10⁻³ m⁴, ρ (density)=2405 kg/m³, E=2.876×10¹⁰ N/m². The results found for the natural frequencies are compared with those found in the literature (Li and Choo, 1984) in Table 1. The problem is solved by SAP2000 package employing the equivalent frame method.

Method		Li and Choo	Galerkin	Matrix progression	SAP2000	Present work
Frequencies (Hz)	Mode 1	2.08	2.08	2.05	2.08	2.08
	Mode 2	9.34	9.37	8.78	9.34	9.34

Table 1. Comparison of the natural frequencies in example 1

Example 2 treats a coupled shear wall with five bays, for which the geometric properties are seen in Figure 3. The physical properties of the wall are as follows: $E=2x10^7$ kN/m², $\rho = 24$ kN/m³, $K_v = 1.3x10^7$ kN/m, $K_r = 4x10^8$ kN-m/rad, $K_h = \infty$. The properties pertaining to all bays are same and the thickness of the wall is 0.3 m everywhere.



Figure 3. Five bay coupled shear wall

The five bay coupled shear wall was solved, first, without stiffeners, and then, with two stiffeners, one at ¹/₄ and the other at ³/₄ of the total height. Free vibration analyses of both cases were carried out both by the present method and by SAP2000 structural analysis program and the first ten natural frequencies (NF) were presented in Table 2, together with their percentage differences. The elastic foundation is modeled by three equivalent springs, one horizontal, one vertical and one rotational, at the bottom of each and every pier.

	V	Without stiffener	s	With stiffeners		
Mode	SAP2000	Present work	Difference	SAP2000	Present work	Difference
	(Hz)	(Hz)	(%)	(Hz)	(Hz)	(%)
1	3.7748	3.7292	1.20	4.0962	4.0930	0.07
2	13.2705	13.0638	1.55	15.2508	15.1348	0.76
3	26.7154	26.3619	1.32	27.7071	27.7726	0.23
4	41.7248	41.3214	0.96	43.3061	43.5509	0.56
5	59.8751	59.5064	0.61	70.5726	70.2616	0.44
6	81.1809	80.8492	0.40	92.5581	92.3744	0.19
7	106.1309	105.8253	0.28	108.4562	111.1260	2.46
8	134.7022	134.3263	0.27	138.5687	139.6644	0.79
9	167.0208	166.4775	0.32	185.1551	183.6118	0.83
10	202.9698	202.0851	0.43	219.0900	218.3422	0.34

Table 2. Comparison of the natural frequencies of the two methods for the five bay coupled shear wall

Conclusions

In the first example, the problem of the free vibration analysis of a single bay coupled shear wall (Li and Choo, 1984) has been solved both by the present method and SAP2000 package (Wilson, 1997), so as to check the validity of the present method. The results of the two methods and those previously found in the literature (Li and Choo, 1984) are in good agreement. The dynamic analysis of multi-bay coupled shear walls has not been studied previously, at least, to the knowledge of the authors. Hence, as example 2, the free vibration analysis of a five bay coupled shear wall is carried out and the results have been compared with those of SAP2000 package.

In the first example, the results of the present method have matched fairly well with those of previous studies mentioned in the literature (Li and Choo, 1984). In the second example, the five bay coupled shear wall has been solved with and without stiffeners and the first ten natural frequencies have been compared with those found by SAP2000 package. As seen in Table 2, the results are in fair agreement.

The method proposed in the present study, is two fold advantageous. Firstly, the data preparation is much easier than that of the equivalent frame method. Besides, for quick trials of many different cases of a structure for optimization purposes, new versions of data can be obtained with much less changes compared to other methods. Secondly, the computation time of the present method is much less than those of other methods. The computation time needed to solve a certain problem is about five times less for the present method than that for the finite difference method (Li and Choo, 1984). Considering the foregoing two advantages, the present method can be used effectively for predesign or dimensioning purposes. Once the dimensioning is complete, the final design can be carried out employing a suitable method consistent with the importance of the project in hand and the accuracy desired.

References

Li, G.Q., Choo, B.S., 1984, Free Vibrations of Coupled Shear Walls, *Proceedings of Fifth International Conference on Advances in Structural Dynamics*, Vol. 2, pp. 859-868.

Meirovitch, L., 1980, *Computational Methods in Structural Dynamics* (First Edition), Netherlands, Sijthoff and Nordhoff.

Wilson, E.L., 1997, SAP2000 Integrated Finite Element Analysis and Design of Structures, California, 1-2, Computers and Structures, Inc.