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Seismic Analyses of Soil-Structure Interaction Systems by Coupling the Finite Element and the Scaled Boundary Finite Element Methods

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ABSTRACT: A coupled model, based on the finite element method (FEM) and the scaled boundary-finite element method (SBFEM) is presented for seismic analyses of soil-structure interaction (SSI) systems. The FEM is used for modeling the finite region and the structure, and the SBFEM is used for modeling the unbounded medium extending to infinity. These two method are combined by using substructure method. The structures considered are assumed to be resting on the surface of the semi-infinite soil medium. The analyses are done in frequency space, and Fast Fourier Transform algorithm is used for transformation. Simple problems are considered to verify the proposed SSI model. The results obtained by the proposed model are compared with the SSI software results namely SASSI and the literature. It is found that the proposed model can be used reliably and effectively in seismic response of SSI problems.

Keywords: soil-structure interaction; seismic analyses; finite element; impedance; scaled-boundary finite element method

ÖZET: Yapı-zemin etkileşimi (YZE) sistemlerinin, sonlu eleman metoduna (SEM) ve ölçeklendirilmiş-sınır sonlu eleman metoduna (ÖSSEM) dayalı bir birleştirilmiş model kullanılarak sismik analizi sunulmuştur. SEM, yakın bölge ve yapının, ÖSSEM sonsuza uzanan sınırlandırılmamış ortamın modellenmesinde kullanılmıştır. Bu iki metot, altyapı metodu kullanılarak birleştirilmiştir. Ele alınan yapıların, yarı-sonsuz zeminin yüzeyine oturduğu düşünülmüştür. Analizler frekans uzayında yapılmış ve transformasyon için Hızlı Fourier Transformasyon algoritması kullanılmıştır. Önerilen modeli test etmek için basit örnekler çözülmüştür. Önerilen model ile elde edilen sonuçlar, SASSI isimli YZE programı sonuçları ve literatürle karşılaştırılmıştır. Önerilen modelin sismik etki altında YZE problemlerinin analizinde güvenli bir şekilde kullanılabileceği görülmüştür.

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Introduction

The dynamic response of massive structures such as nuclear power plants, dams, liquid-storage tanks and high-rise buildings may be affected by the SSI as well as the dynamic characteristics of the exciting loads (i.e. earthquake, wind, explosion, machinery vibrations) and the structures. The effect of the SSI may be very important especially for stiff, rigid and massive structures resting on relatively soft soil. The effect of the SSI may alter the dynamic characteristics of the structural response significantly. Thus, the interaction effects have to be considered in the dynamic analyses of the structures in a semi-infinite medium (Bettess, 1977; Wolf and Song, 1996; Von Estorff and Kausel, 1989; Yerli et al., 1998; Kim and Yun, 2002; Genes and Kocak, 2002).

Dynamic SSI analyses of structures in a semi-infinite medium can be conducted by using the direct method (DM) and the substructure method (SM). In the SM, the whole or a part of the soil region is represented by impedance (dynamic-stiffness) or compliance coefficients or matrices. The calculated impedance coefficients or matrices of the soil medium are added to the dynamic-stiffness matrix of the finite region or the structure satisfying the equilibrium and continuity conditions. DM and SM have their advantages depending upon the problem to which they are applied. In the literature one can find numerous work related to both of these methods.

In SSI analyses the main challenge is to model the soil medium extending to infinity. In order to consider the effect of an unbounded soil medium, several methods are proposed e.g. transmitting boundary method (Lysmer et al. 1981; Lysmer and Kuhlemeyer, 1969; White et al., 1977), boundary element method (BEM) (Karabalis and Beskos, 1984; Ahmad and Banerjee, 1988; Mengi et al., 1994), infinite element method (Yerli et al., 1998; Medina and Penzien, 1982; Yun et al., 2002), the scaled boundary-finite element method (SBFEM) (Wolf and Song, 1996,2000; Genes and Kocak, 2002; Genes, 2001; Song and Wolf, 1997,2000), the coupled finite element/boundary element method (Qian et al., 1996, Von Estorff and Firuziaan, 2000), and damping-solvent extraction method (Song and Wolf, 1994).

In SMs the effect of the unbounded soil medium is considered by calculating the impedance coefficients or the impedance matrices of the soil. These coefficients or matrices, in general, depend on the geometry of the footing (i.e. circular, rectangular, strip, as at the surface or embedded) and elastic properties of the medium (i.e. uniform and layered soil, with or without hysteretic damping). In order to calculate the impedance coefficients or matrices of various types of soil mediums for different foundations, one of the models namely SBFEM was proposed by Wolf and Song (1996 and 2000) and Song and Wolf (1997 and 2000). In these studies, the formulation of the SBFEM had been given and the method verified by calculating the impedance coefficients of different types of rigid footings for uniform and layered soil mediums in time and frequency spaces. In the study of Genes and Kocak (2002), SBFEM also had been applied to the embedded and surface structures with uniform and layered soil mediums under harmonic and transient loadings. Also, in the study of Genes and Kocak (2002), the calculation of the impedance matrices of the soil medium for large-scale problems had been done on the parallel platforms. In that study, significant computation time was saved by using multiple processors concurrently.

To our knowledge, the SBFEM has not been applied to earthquake analysis in the literature yet. The objective of this study is to develop a model by coupling the FEM with the SBFEM to analyses two- and three-dimensional (2D and 3D) structures under seismic excitation. In the literature, various models are proposed for the analyses of SSI problems under seismic excitation, e.g. Luco and Wong, 1987, Caner and Mengi, 1997, and Choi, et al., 2001.

In this study, the analyses are done in frequency domain, and Fast Fourier Transform (FFT) is used for the transformation between frequency and time space, (Cooley, et al., 1969). To assess the proposed SSI model, simple problems are chosen, i.e. a single plate resting on a rigid strip foundation and a single column resting on a square foundation. The results obtained by the proposed model are compared with the SSI software, SASSI (Lysmer, et al., 1981), and the literature. It is found that the proposed model can be used reliably in seismic response of SSI problems.

Equation of Motion

The methods used for the coupled model to represent the soil and the structure as well as the solved systems of equations are briefly described in the following.

SBFEM, proposed by Wolf and Song (1996, 1997), is based on the calculation of impedance matrix defined on the truncated soil region which extends to infinity. The computer program, SIMILAR, is also available to calculate only the impedance matrix on the interface of the near- and the far-field for elastic soil medium. In a previous work by Genes and Kocak (2002), a computer program based on SBFEM was presented for the analyses of SSI systems under harmonic and transient loadings for elastic and viscoelastic soil medium. Here, our main focus is the inclusion of seismic effects in the model.

In order to analyse structures under seismic excitation, the impedance matrices of the soil extending to infinity must be calculated. In this study the impedance matrices are calculated by integrating the first-order non-linear ordinary differential equation (ODE) resulting from SBFEM formulation (Wolf and Song, 1996, 1997). The SBFEM can calculate the impedance matrix at the structure-medium interface without requiring any discretization of the soil part by finite elements (FEs) for embedded structures. Due to the definition of the SBFEM, for surface structures the interface between the structure and the soil medium can not be modeled explicitly by SBFEM. Therefore, an impedance analysis is required for the calculation of the impedance matrix at the structure-medium interface. Thus, the soil is discretized by one row of FEs as shown in Fig. 1 and the interface between the near- and far-field is discretized and modeled by SBFEM for uniform half-space. The impedance matrix calculated by SBFEM is then added to the soil satisfying the equilibrium and continuity conditions. The resulting equation can be written as,

$$\begin{Bmatrix} \mathbf{F}_b \\ \mathbf{F}_a \end{Bmatrix} = \begin{bmatrix} \mathbf{S}_{bb} & \mathbf{S}_{ba} \\ \mathbf{S}_{ab} & \mathbf{S}_{aa} + \mathbf{S}_s^{\infty} \end{bmatrix} \begin{Bmatrix} \mathbf{U}_b \\ \mathbf{U}_a \end{Bmatrix} \tag{1}$$

where, **F**, **U** and **S** designate the system load, displacement vectors, and dynamic-stiffness matrices of the soil (near-field modeled by FEM, see Fig. 1), respectively. The subscripts *a* and *b* designate the interface and non-interface nodes of the FE mesh representing the soil. $\mathbf{S}_s^\infty$ is the impedance matrix of the far-field calculated by SBFEM. The interaction nodes on the half-space (see Fig. 1) are loaded by unit load for all degrees of freedoms sequentially in frequency space to calculate the columns of the compliance matrix for each frequency. For example, for an interface with N numbers of interaction nodes, Eq.(1) have to be solved 2N times for a compliance matrix of a two-dimensional problem for a specific frequency. The inverse of the compliance matrix results in impedance matrix ($\mathbf{S}^\infty$) of the semi-infinite soil medium at the interaction nodes. In this study, the impedance matrices of the soil medium for the frequencies between the interval from zero to cutoff frequency are calculated first and stored for later calculations using SM. Then, the structure is analysed as mentioned in the following paragraphs.

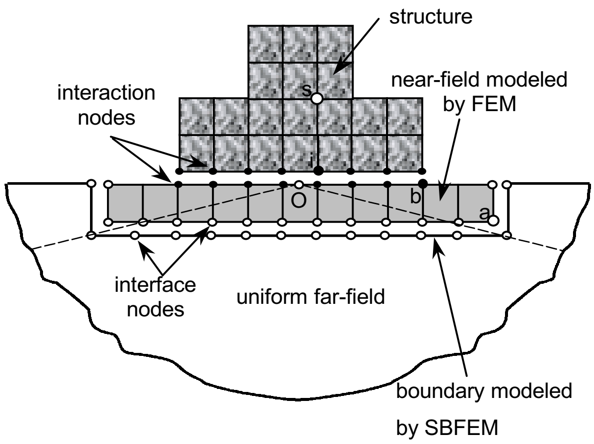

Figure 1. Concept of FEM with SBFEM for SSI analysis under seismic response

The dynamic-stiffness matrix of the structure can be calculated by standard FE formulation. The equation of motion of the structure can be written as in Eq.(2)

$$\begin{Bmatrix} \mathbf{F}_s \\ \mathbf{F}_i \end{Bmatrix} = \begin{bmatrix} \mathbf{S}_{ss} & \mathbf{S}_{si} \\ \mathbf{S}_{is} & \mathbf{S}_{ii} \end{bmatrix} \begin{Bmatrix} \mathbf{U}_s \\ \mathbf{U}_i \end{Bmatrix} \tag{2}$$

where, the subcripts *s* and *i* designate the interaction and non-interaction nodes of the structure. When there is no seismic excitation, $\mathbf{F}_i$ is given as in Eq.(3).

$$\mathbf{F}_i = -\mathbf{S}^\infty \mathbf{U}_i \tag{3}$$

Substituting Eq.(3) into Eq.(2) results an equation similar to Eq.(1) only in the change of subscripts *a* and *b* to *i* and *s* for interface and non-interface nodes of the structure, respectively (see Fig. 1), and also the replacement of $\mathbf{S}_s^\infty$ by $\mathbf{S}^\infty$. When there is seismic excitation, Eq.(3) should be modified so that we satisfy the condition that when the structure is removed, the resulting forces and displacements along the interaction surface must be the same as those of free field. This condition can be satisfied by modifying Eq.(3) as,

$$\mathbf{F}_i = -\mathbf{S}^{\infty}\left(\mathbf{U}_i - \mathbf{U}_i^{ff}\right) \tag{4}$$

In Eq.(4), $\mathbf{U}_i^{ff}$ is the displacement vector describing the translations and relations of the footing under the influence of seismic input and is related to the control point motion by an equation of $\mathbf{U}_i^{ff} = \mathbf{T}\mathbf{U}_c^{ff}$, where $\mathbf{U}_c^{ff}$ is a vector including the free field displacements recorded at the control point, and $\mathbf{T}$ is a transfer matrix called earthquake input motion matrix (Wong and Luco, 1978). The superscript *ff* designates the free field. The results in Fourier transform space may be inverted through the use of FFT (Cooley et al., 1969) algorithm to find the corresponding solution in time space**.** By inserting Eq.(4) into Eq.(2), and by equating sub-load vector of the non-interaction nodes of the structure (F_s) to zero results in Eq.(5). Eq.(5) represents the equation of motion of a structure under the effect of seismic excitation.

$$\begin{Bmatrix} \mathbf{F}_s(=0) \\ \mathbf{S}^{\infty}\mathbf{U}_i^{ff} \end{Bmatrix} = \begin{bmatrix} \mathbf{S}_{ss} & \mathbf{S}_{si} \\ \mathbf{S}_{is} & \mathbf{S}_{ii} + \mathbf{S}^{\infty} \end{bmatrix} \begin{Bmatrix} \mathbf{U}_s \\ \mathbf{U}_i \end{Bmatrix} \tag{5}$$

The above calculated and used impedance matrices of the soil might not be calculated at the Fourier frequencies. This situation results from the integration of the ODE of SBFEM with Bulirsch-Stoer integration algorithm (Bulirsch and Stoer, 1966). The algorithm self controls the steps of the independent frequency automatically. Thus, the resulting independent frequencies might not be overlapped with the Fourier frequencies. Therefore, the used impedance matrices are selected according to the frequencies which are nearest to the Fourier frequencies.

Numerical Results and Discussions

In this section, some example problems are solved under the effect of seismic excitation by the present SSI model. The results are presented and compared with the computer program SASSI and the literature. In the present model, infinite soil region is defined by three-node line and eight-node surface elements in 2D and 3D cases, respectively. Also, the finite region is defined by eight-node surface elements and twenty-node prismatic elements in 2D and 3D cases, respectively. In all of the examples, the control point motion prescribed is the acceleration record of S16E component of San Fernando Earthquake recorded at Pacoima Dam, Cal. in 1971. The time increment of this record is Δt=0.02 sec. In all of the computations, non-dimensional (ND) space is used. Thus, the adjusted ND time increment is chosen to be $\Delta\bar{t} = 1$. The transformation to ND space is obtained by choosing the following characteristic quantities: for the force; $f = Ga^2$; for the time; $\tau = a / c_s$; and for the length, $l=a$ where G is shear modulus of the soil, c_s is shear wave velocity in the soil and a is half width of the foundations.

Elastic Plate Resting on a Rigid Strip Foundation

A simplified system which is presented as a massless rigid strip foundation and a plate resting on it with lumped mass (see Fig. 2) is analysed. The strip foundation of width 2a is assumed to be resting on an elastic layer of thickness d=10a. The ND quantities of the system are considered as $\overline{a}=1$, $\overline{L}=1$, $\overline{d}=10$, $\overline{m}=1$, $\overline{G}=1$, $\overline{\rho}=1$ and $\nu=0.2$ where L, m, ρ and ν are the length of the plate, mass of the lumped mass, unit weight and Poisson's ratio of the soil, respectively. This system is treated as a plane-strain problem. The seismic input is generated by vertical SV waves of Pacoima Earthquake, whose motion is polarized in the horizontal direction, which implies that the free field displacements on planes parallel to the top surface are uniform. Thus, the only free field displacement is translation on the free surface of the layer. The control point is located in the site of the foundation. When the control point is in the site, the free field motion in horizontal direction simply equals to the control point motion.

In the analyses the soil layer is truncated in the horizontal direction and the remaining medium extending to infinity is modeled by SBFEM. The finite region bounded by the SBFEM surfaces, rigid bedrock and soil surface is discretized by eight-node FEs (see Fig. 2). As mentioned in the previous section two methods are combined (see Eq.(1)) for the impedance analysis to calculate the impedance matrices at the interface between the strip foundation and the soil layer. The calculated impedance matrices are stored for later usage. Then, the structure is modeled by the FEs and analysed to calculate the ND maximum shear force at the lower end of the plate for different bending rigidities. The ND shear force is calculated as $\overline{V}=V/Ga^2$. The same problem is solved by the SSI analysis program SASSI. The results are compared with the results computed by Tanrikulu and Yerli (1995). Tanrikulu and Yerli (1995) solved the problem by using BEM for the calculation of impedance coefficients. Studying the Fig. 3, one can see that the present model gives very close results to the others. Also, from the examination of Fig. 3, the maximum shear force is high for low bending rigidities of the plate. In high bending rigidities, the change in maximum shear force does not change significantly.

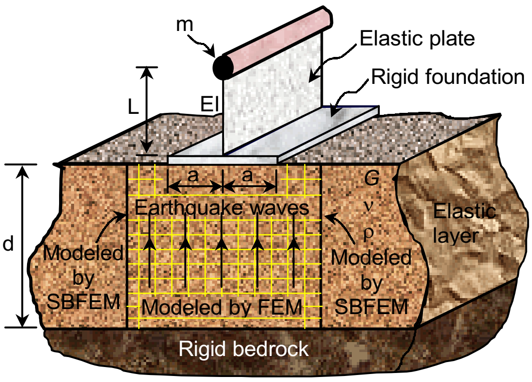

Figure 2. Elastic plate resting on a rigid strip foundation

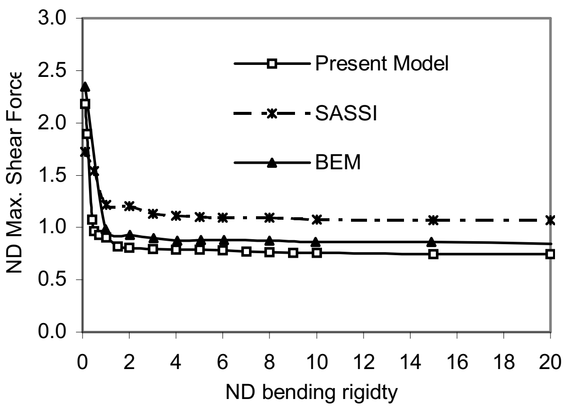

Figure 3. Maximum shear force via bending rigidity

Elastic Column Resting on a Rigid Square Foundation

A simplified system composed of a massless rigid square foundation and a column resting on it with lumped mass (see Fig. 4) is analysed. The square foundation rests on elastic half-space. The ND quantities of the system are considered as $\overline{a}=1$, $\overline{L}=1$,

$\overline{m}=1$, $\overline{G}=1$, $\overline{\rho}=1$ and $\nu=1/3$. The seismic input is generated by vertical SV waves of Pacoima Earthquake, whose motion is polarized in the x-direction and the control point is located at the origin of the coordinate system shown in Fig. 4. As in the previous example, the only free field displacement is translation on the free surface of the half-space, and the free field motion in x-direction simply equals to the control point motion.

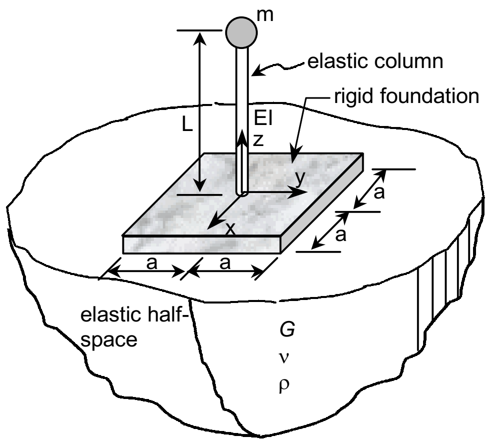

Figure 4. Elastic column resting on a rigid square foundation

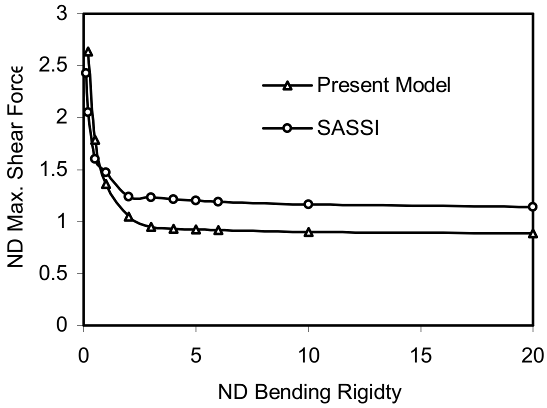

Figure 5. Maximum shear force via bending rigidity

First, the half-space is discretized with one layer twenty-node prismatic FEs and the remaining medium extending to infinity is modeled by SBFEM (consider the Fig. 1 in three dimensions). As mentioned in the previous section, two methods are combined for the impedance analysis to calculate the impedance matrices at the interface between the square foundation and the soil medium by using Eq.(1). By following the same steps described in the previous example, the ND maximum shear force values at the lower end of the column are calculated for different bending rigidities. The ND shear force is calculated as $\overline{V}=V/Ga^2$. Also, the ND accelerations are calculated at the upper end of the column for ND bending rigidities for $\overline{EI}=1$ and $\overline{EI}=5$. The same problem is solved by the SSI analysis program SASSI. The results are compared in Fig. 5 for the maximum shear forces. The accelerations calculated at the upper end of the column are compared in Fig. 6.

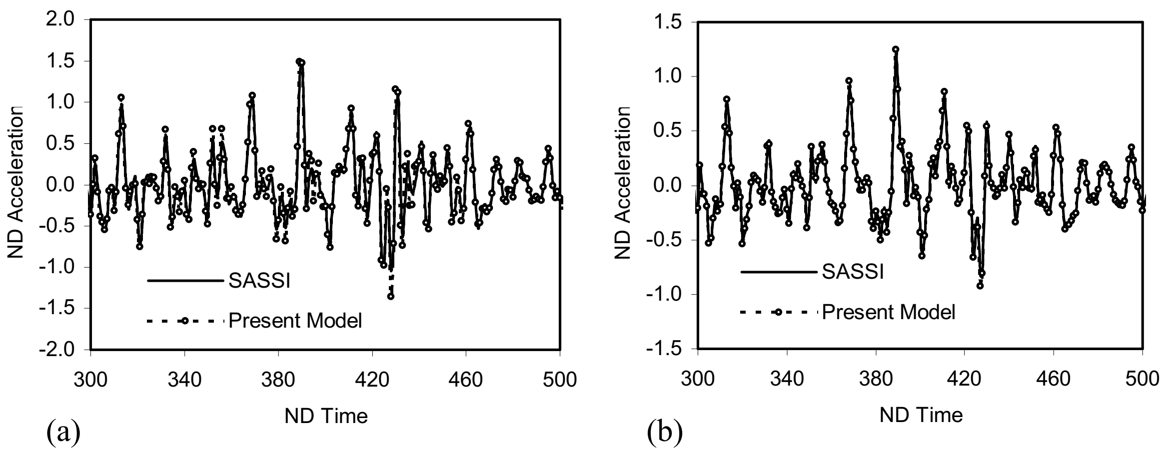

Figure 6. ND accelerations at upper end of the column, a) ND *EI*=1, b) ND *EI*=5

Studying Fig. 5, one can see that the present model gives results close to SASSI. By examining Fig. 5, similar observation can be made for the variation of the maximum shear force (at the lower end of the column) versus bending rigidity. From Fig. 6(a) and 6(b), one can see that the calculated accelerations by the present model at the upper end of the column are very close to the accelerations calculated by SASSI.

Conclusions

A Coupled FEM-SBFEM SSI model for seismic excitation is presented. In the coupled model, SBFEM is used for infinite soil region, and finite soil region, which is modeled by FEM, is added on top. The structures considered are assumed to be resting on the surface of the semi-infinite soil medium. Thus, for surface structures, impedance analysis is required. Two example problems are presented to validate and investigate the applicability and reliability of the model to the structures under seismic excitation. The results of the present model are compared with the literature and SSI analysis program SASSI, and a good match is noted.

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